# Mathematical Methods in Engineering and Applied Science Problem Set 6.

Kovalev Vyacheslav

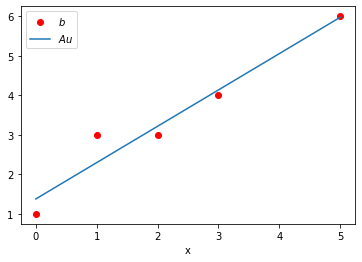
1. Given the data:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 0 | 1 | 2 | 3 | 5 |
|  | 1 | 3 | 3 | 4 | 6 |

1. Find the best linear fit by solving the normal system by hand , with and . Plot the data and the fit.

;

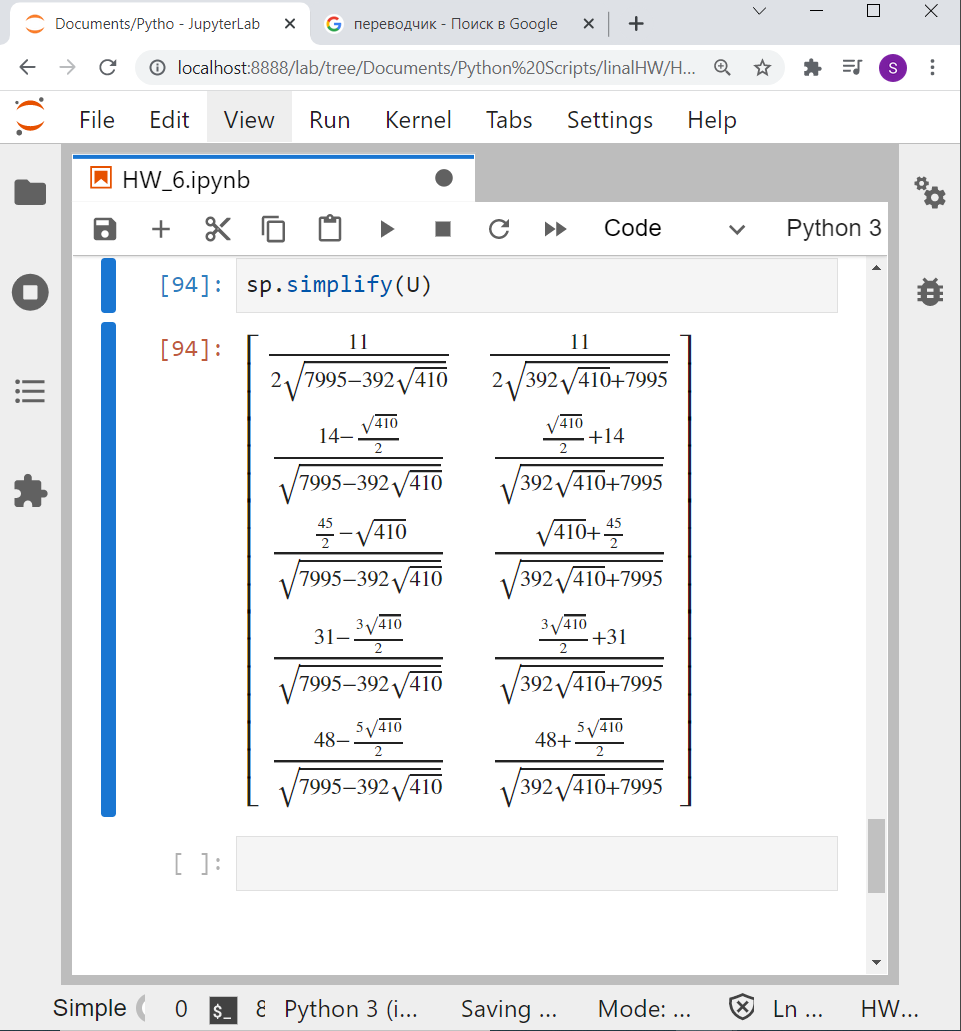
=>

  
  
Data and the fit:

1. Calculate the Moore-Penrose pseudo-inverse of directly from its definition.
2. Write down the SVD of .

Let’s find

=> ; omit process of searching e-values, e-vectors because of difficulties.



– is SVD of , all matrices obtained above.

1. What is the error vector of the approximation and its 2-norm?
2. Изображение выглядит как текст

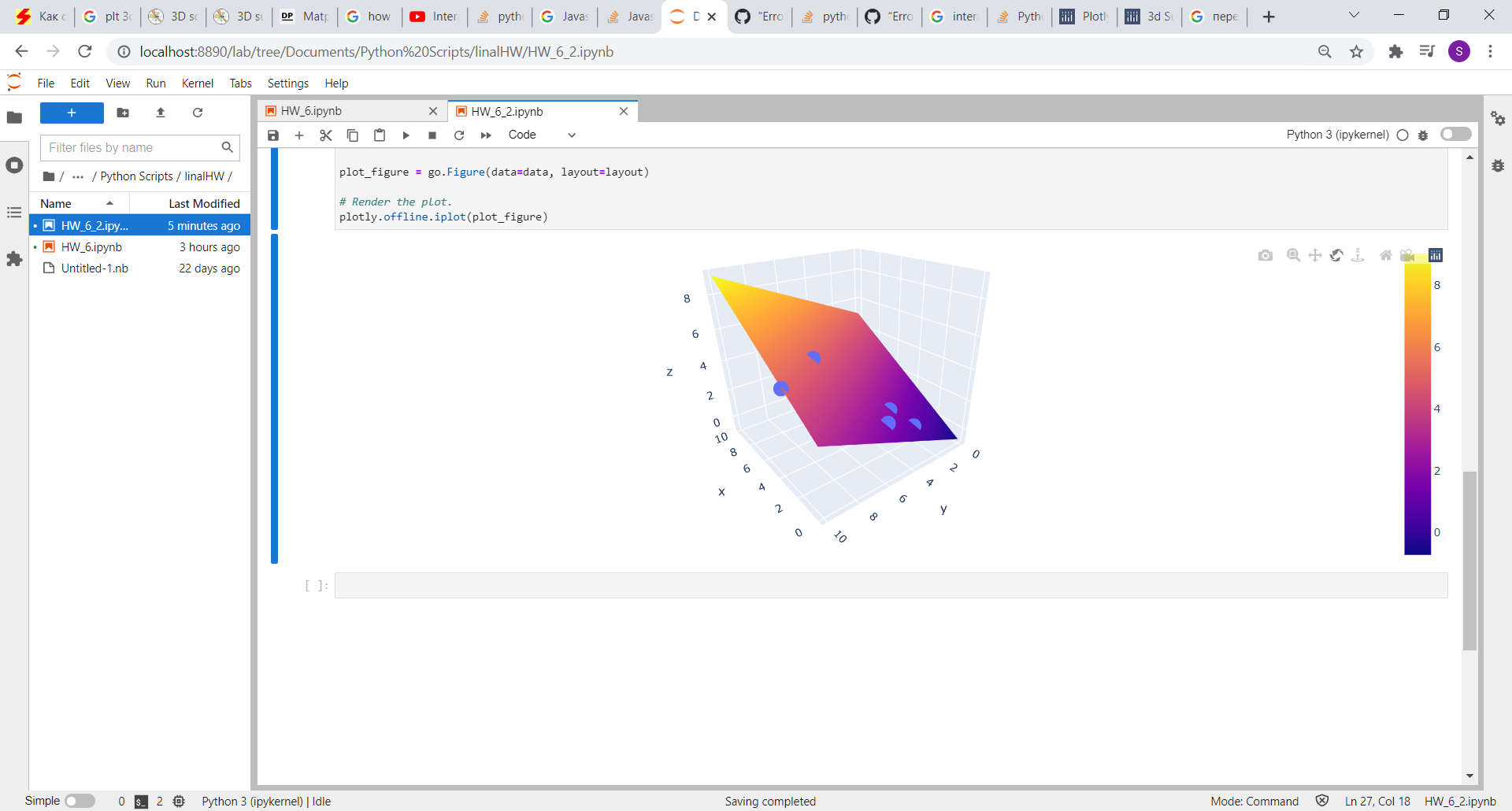
   Автоматически созданное описаниеFind the best plane in , in the least-squares sense, through the data given in the table:   
   What is the error vector and its norm?

Plane equation in terms of matrices:  
; where are columns.

Изображение выглядит как текст

Автоматически созданное описание

where following the same procedure as in previous task obtain:

  
Finally:

Blue points are data points.

Plane is constructed by for

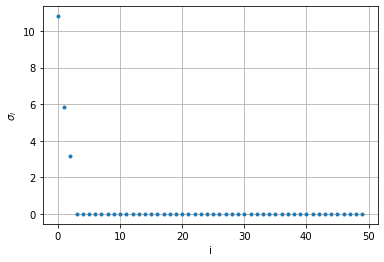
1. Determine the dominant modes in the functionfof spacexand timet:

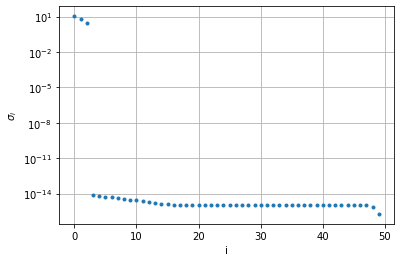
, considering the interval

1. Plot the singular values in uniform as well as semilog scales.  
    Where

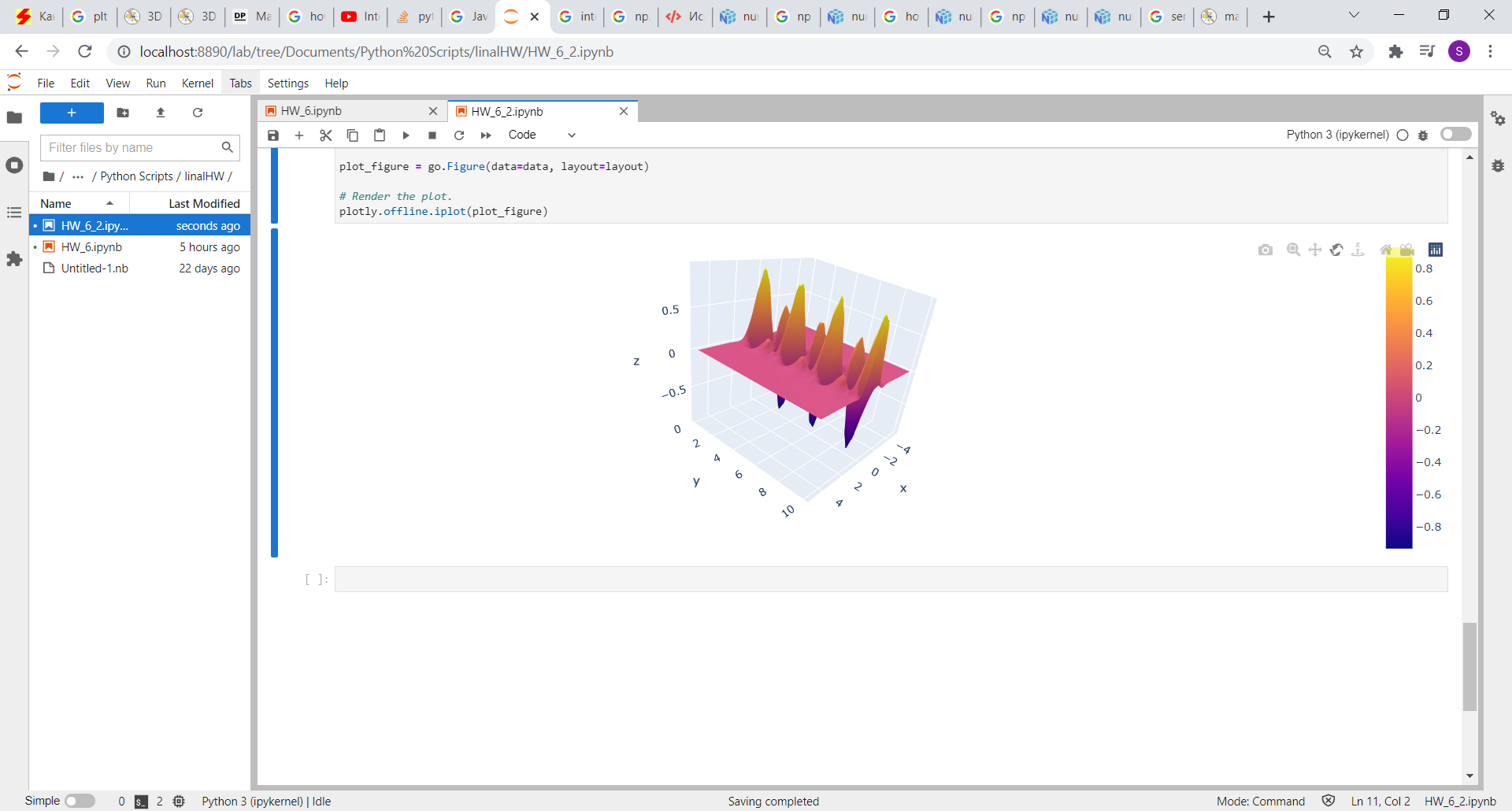
Though we got

Uniform:

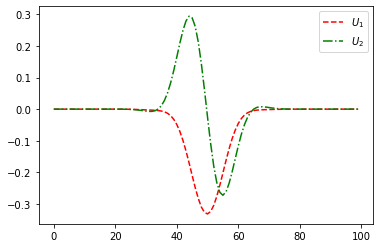
  
Semilog



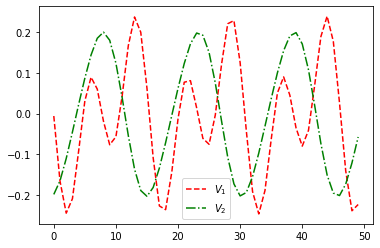
1. Plot the solution in the plane over the given interval.



1. How much “energy” of the solution is contained in mode 1 and in modes 1+2?
2. Plot the first two columns of and in the SVD of matrix obtained by calculat-ing over a grid with 100 points in and 50 points in . Explain their meaning.



The columns of are identified as spatial modes while components of contain the time evolution of the modes. The singular value measure how much of the mode contributes to the “energy” of the signal by .



1. To find a root of , Newton’s method tells to start with some initial guess and then to iterate following the scheme:
2. Use this method to find the root of .

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
| 2 |  |  |  |  |

1. hat is the range of initial conditions that give convergence to ?

First, we can’t divide by 0 => ;

Second, we can’t divide by 0 => let’s find that leads to 0

But is real => no one leads to 0

Third, if

After solving the system obtain:

- but it is the solution of

Or we get solution for from complex space which is impossible.

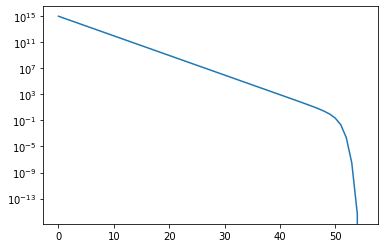
Now we converge system to 1. But as you can see , so if then as sum of positive values. We want to => . We have only one positive point to converge ( ), that means our system will converge to it (can’t converge to , because ).

Finally, .

1. How fast do the iterations converge? Plot the error as a function of (may be, in log scale).

Suppose there is typo in the task, because solution of is , when the error must be 0 but we get

Let .

Go from on the chart:

Suppose:

divide by

after taking log we get for :

for near k and p I got – the order of convergence. So, method converges very fast.

1. Now apply the same Newton iterations as in the previous problem to the equation . Clearly, this equation has no real roots.
2. The question is: What do the iterations do? Do they converge to anything?

. Where => doesn’t converge to anything.

; ;

Though will lead to .

It means that aims to fall to , oscillate around 0.

What if .

So, I think where is will oscillations, till finally becomes and next in iteration will division by 0, the end.

1. How does the behavior of the iterations depend on the initial point?

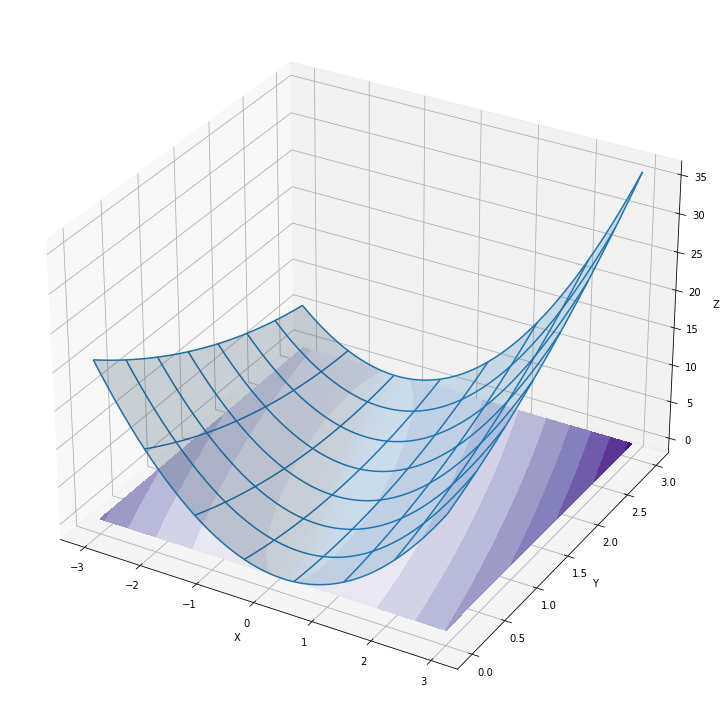
If it starts from it breaks. If it starts from leading to 0 it breaks (and others which leads to ).

Otherwise aims to fall to , oscillate around 0

1. What if you start the iterations in the complex plane? Can you get convergence to the actual roots of the equation? What are the domains of attraction of the roots?

Yes, we can get convergence to the actual roots if start the iterations in the complex plane. .

Yellow is place of complex space where converges to Purple, where converges to - domains of attraction of the roots.

1. Consider the function .
2. **Find its minimum analytically by representing as . Plot the functiontogether with its contour levels using, for example, function in Matlab.

where

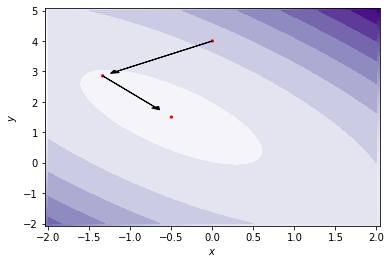
1. Now find the minimum using the gradient descent. Determine the step in the descent method.

Find such .

NOTE: In my case I’ve found as a row, therefore .

Using the algorithm found , ; almost always equals to .

1. Starting with , calculate the first two steps of the gradient descent explicitly and indicate on a single plot both the positions and the gradient vectors at those positions. Also plot the level curves off going through these points.

Let’s

1. Implement the descent algorithm in Matlab or Python and starting with the same initial condition as in (c) find the minimum within a tolerance of . How many iterations does it take to reach the minimum?

iterations to get: ;